FERMI PROBLEMS in primary mathematics classrooms

Fostering children's mathematical modelling processes

ANDREA PETER-KOOP

explains how children in some German classrooms developed problem solving processes through working on challenging open style problems.

The difficulties which primary students experience when dealing with real-world related word problems have been discussed extensively. These difficulties are not only related to complex, non-routine problems but already occur with respect to routine problems that involve the application of a simple algorithm. Due to difficulties with the comprehension of the text and the identification of the 'mathematical core' of the problem, primary students frequently engage in a rather arbitrary and random operational combination of the numbers given in the text. In doing so, they fail to acknowledge the relationship between the given data and the real-world context. Real-world problem solving involves the 'mathematisation' of a non-mathematical situation that involves:

- the construction of a mathematical model with respect to the real-world situation,
- the finding (calculation) of the unknown, and
- the transfer of the mathematical result derived from the mathematical model to the real-world situation.

Hence, this 'mathematisation' process is frequently modelled itself in the manner shown in Figure 1.

However, while traditional word problems often do not seem to provide a suitable context for the development of mathematical modelling skills, the use of Fermi problems in the middle and upper primary mathematics classroom can help to foster students' mathematical modelling strategies.

Fermi problems

Enrico Fermi (1901–1954), who in 1938 won the Nobel Prize for physics, was known by his students for posing open problems that could only be solved by giving a reasonable estimate. Fermi problems such as, 'How many piano tuners are there in Chicago?' share the characteristic that the initial response of the problem solver is that the problem could not possibly be solved without recourse to further reference material.

Enrico Fermi

However, while individuals frequently reject these problems as too difficult, Clarke and McDonough (1989) pointed out that' 'pupils, working in cooperative groups, come to see that the knowledge and processes to solve the problem already reside within the group' (p. 22). In order to stimulate collaborative modelling strategies while avoiding frustration by setting too demanding tasks, Fermi problems suitable for middle and upper primary students should:

- be based on a selection of real-world related situations that include reference contexts for primary students;
- present challenges and intrinsically motivate cooperation with peers;
- be open-beginning as well as open-ended real-world related tasks that require decision making with respect to the modelling process;
- not contain numbers in order to challenge students to engage in estimation and rough calculation and/or the collection of relevant data.

The following Fermi problems have been successfully used in grade 3 and 4 classrooms:

How much paper does your school use in one month? (paper problem)

How many children are together as heavy as a polar bear? (polar bear problem)

How much water do you use in one week? (water problem)

There is a 3 km tailback on the motorway. How many vehicles are caught in this traffic jam? (traffic problem)

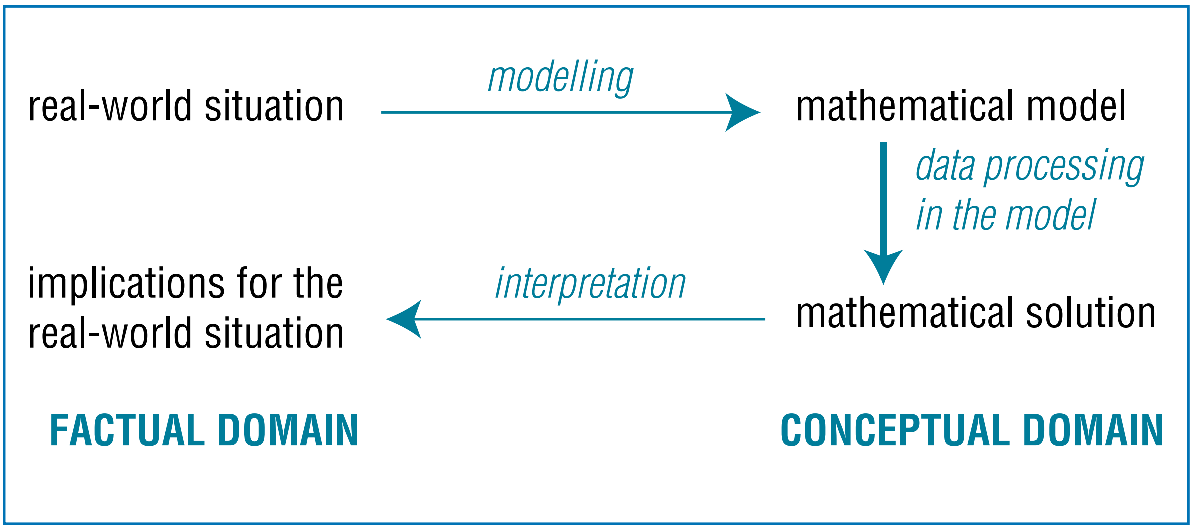


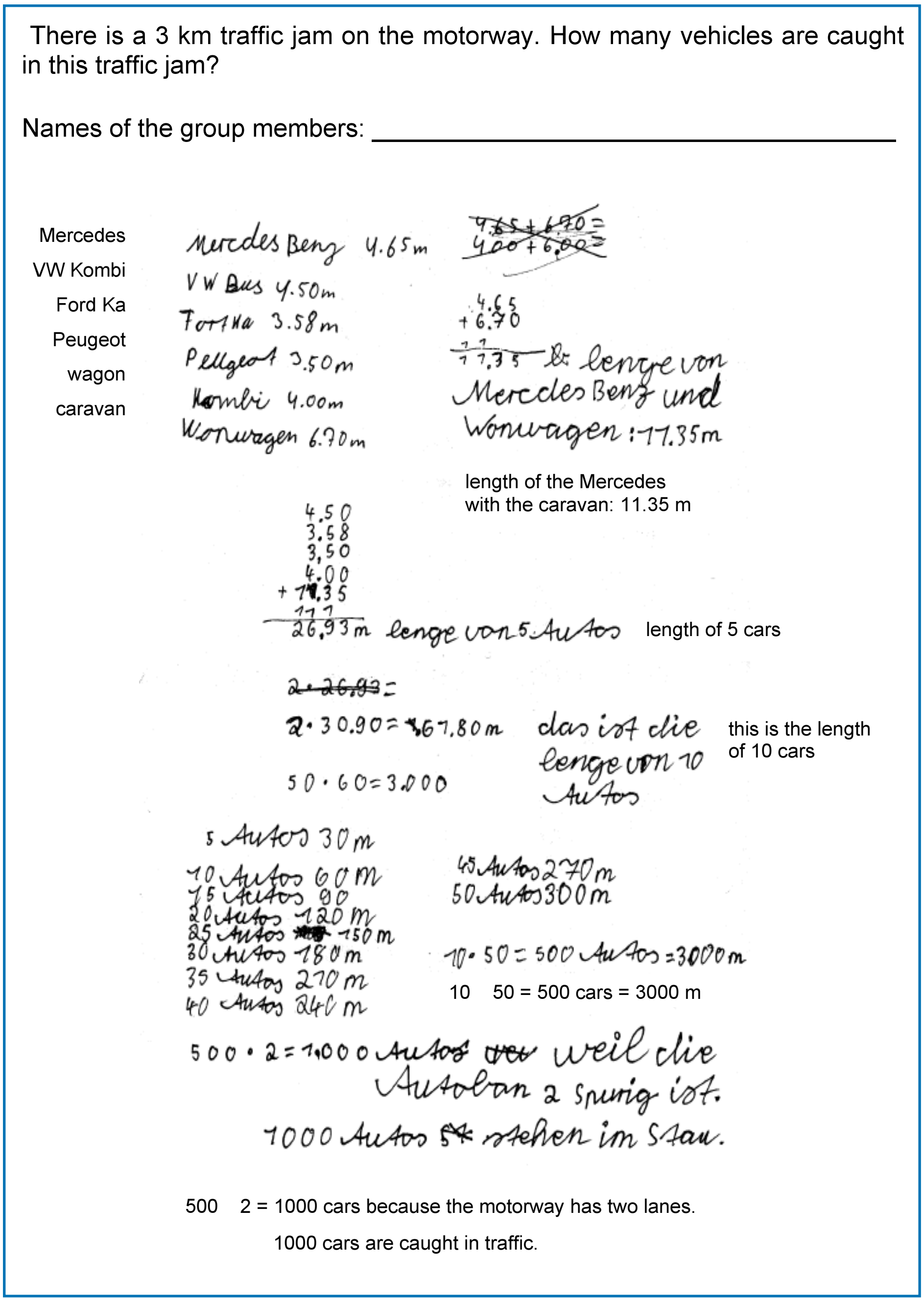
Figure 1

Introducing the problem

Allow a brief first round of discussion before splitting the class in small groups in order to make sure that all students have understood the problem. Have equipment and literature for data collection (such as scales, information on bus/train fares, books on animals, measuring tapes, etc.) available for immediate use according to specific requests by the students. Consider that enough time is allowed for the group work. In the German classrooms the groups needed between 30 to 60 minutes in order to find a satisfactory solution. In some cases about half of this time was needed for the data collection and background research.

Make it clear to the groups that their work will be presented to the whole class in a 'strategy conference' once all groups have finished their work and that each group member is expected to describe and explain the group's solution. The group work is finished when all group members are satisfied with the solution and able to present the work. Ask the children to use the provided worksheet (see Worksheet 1 in Figure 2) to write down their work. An overhead transparency copy of the worksheet helps the students to explain their work as well as to relate to the results and strategies of the other groups.

Encourage the children to generate tables, make drawings, or do little experiments in order to foster powerful mathematical problem solving strategies. The drawing shown in Figure 3, from a mixed ability group of fourth


There is a 3 km traffic jam on the motorway. How many vehicles are caught in this traffic jam?

Names of the group members: ______________________

Mercedes — Mercedes Benz 4.65 m
VW Kombi — VW Bus 4.50 m
Ford Ka — Ford Ka 3.58 m
Peugeot — Peugeot 3.50 m
wagon — Kombi 4.00 m
caravan — Wonwagen 6.70 m

~~4.65 + 6.70 =~~
~~4.00 + 6.00 =~~

4.65 + 6.70 = 11.35 die lenge von Mercedes Benz und Wonwagen: 11.35 m

length of the Mercedes with the caravan: 11.35 m

4.50 + 3.58 + 3.50 + 4.00 + 11.35 = 26.93 m lenge von 5 Autos — length of 5 cars

~~2 · 26.93 =~~
2 · 30.90 = 61.80 m das ist die lenge von 10 Autos — this is the length of 10 cars

50 · 60 = 3.000

5 Autos 30 m
10 Autos 60 m
15 Autos 90
20 Autos 120 m
25 Autos 150 m
30 Autos 180 m
35 Autos 210 m
40 Autos 240 m
45 Autos 270 m
50 Autos 300 m

10 · 50 = 500 Autos = 3000 m
10 50 = 500 cars = 3000 m

500 · 2 = 1.000 Autos weil die Autoban 2 spurig ist. 1000 Autos stehen im Stau.

500 2 = 1000 cars because the motorway has two lanes.

1000 cars are caught in traffic.

Figure 2. Worksheet showing one group's strategy.

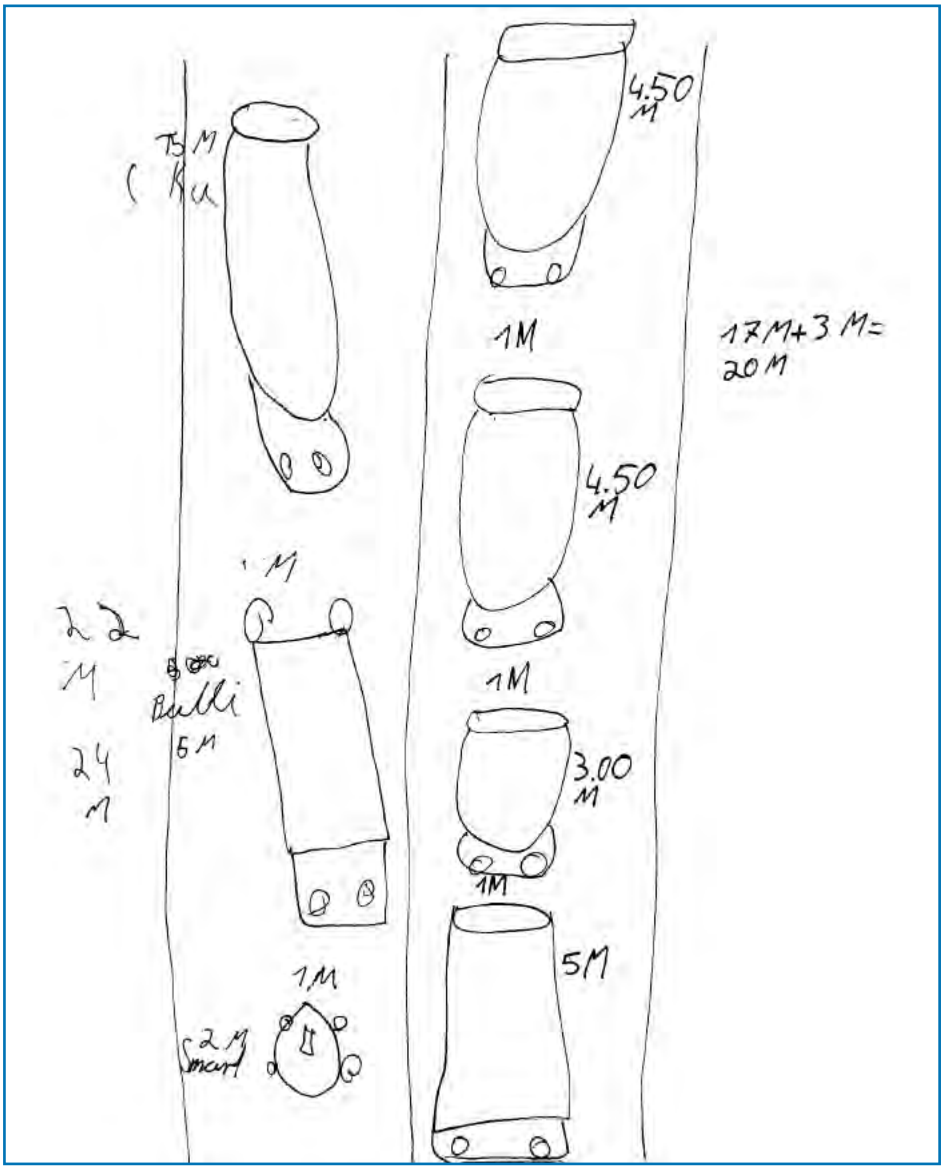


Figure 3. Drawing showing 1 metre separation of vehicles.

Figure 4. Collecting authentic data.

graders, shows their illustration of the traffic jam acknowledging a 1 metre separation gap between the vehicles.

Figure 2 shows an example of a group work sheet. The solution of four supposedly low achieving fourth grade boys of the traffic problem has been included to illustrate what ideas and strategies children might develop and employ. The first part of the group work was dedicated to measuring the lengths of different vehicles in the school car park and the direct school neighbourhood (see Figure 4). In our experience most groups preferred to work with authentic data rather than basing their work on estimations.

Mathematical modelling processes

Observations in the German classrooms suggest that, in general, the students did not develop and then execute a solution plan. On the contrary, frequently a rather aimless and unstructured approach was observed. While most groups, including the low achievers, were generally highly successful in finding an appropriate solution, the mathematical modelling process leading to that solution was determined by a slowly developing process in which hypotheses were generated, tested, confirmed or neglected. Meanwhile, arithmetic results were interpreted, leading to the development of further solution ideas. However, while the literature suggests only one modelling cycle

(as already presented), in most cases the different stages of the modelling processes were revisited several times, hence leading to a multi-cyclic structure.

Furthermore, in many cases the outcome of the modelling activity was a conceptual tool that exceeds the solution of the specific problem. For example, the working as well as the presentation of the work during the strategy conference suggests that the four boys who developed the solution shown on the worksheet (Figure 2) have discovered the concept of proportionality. They achieved this by using proportional calculation in order to determine the number of cars in a 3 km traffic jam, based on their estimate that 5 cars occupy approximately 30 m. They also used this same strategy later, when dealing with the polar bear problem.

Strategy conferences and assessment

It is interesting to note that young children obviously prefer what seems to be an unstructured approach which often only during the presentation of their work in whole class discussions becomes systematic. Hence, the strategy conferences provided substantial opportunity for the students to reflect on and compare their work with other approaches. In this context, the fact that there was always more than one reasonable and acceptable solution only initially presented the children with irritation and led to controversial discussions. After that, they demonstrated increasing confidence and enjoyment with respect to the challenges set by the different problems.

While, traditionally, the assessment of the problem solution is seen as the responsibility of the teacher, the strategy conferences also provide a suitable forum for peer assessment. In this context each student has two roles: on the one hand as a group member he/she shares responsibility regarding the appropriate solution of the given task and its presentation. On the other hand, each student also acts as a critical and informed evaluator with respect to the work of the other groups and hence has to argue from both perspectives. In our observations, this led to friendly and constructive criticism while successful solutions and innovative approaches were honoured with praise.

Reference

Clarke, D. J. & McDonough, A. (1989). The problems of the problem solving classroom. *The Australian Mathematics Teacher, 45* (2), 20–24.

Prof. Dr Andrea Peter-Koop
University of Oldenburg, Germany
<peter-koop@mathematik.uni-oldenburg.de>

APMC